Chapter 3 · The World of Numbers

WORK IT OUT

A maths workbook — concepts, worked examples, graded practice and number puzzles

Do the Learn First pages before the practice. Do the practice before the puzzles. Full solutions are at the back — but they only help you after you have tried.

Part	What it is	How to use it	Time
1	Learn First	Read and annotate. Copy every worked example by hand.	25 min
2	Worked Examples	Cover the solution. Try. Then compare line by line.	20 min
3	Guided Practice	Steps are given; you fill the gaps.	15 min
4	Practice Set A · Fluency	Straight application. Aim for speed and accuracy.	20 min
5	Practice Set B · Reasoning	Multi-step. Show every line.	25 min
6	Practice Set C · Challenge	Stretch problems. One may beat you — that is fine.	25 min
7	The Crossnumber	Answers are numbers. Wrong maths will not fit.	20 min
8	Number Search & Magic Square	Solo or pairs.	15 min
9	Spot the Mistake	Audit a worked solution. Find the planted error.	15 min
10	Quick-Fire Round	Matching, True/False, Two Truths and a Lie.	15 min
11	The Escape Challenge	Teams of 4.	25 min

Part 1 · Learn First

Long before there were equations on a board, there was a herder counting cattle with pebbles. This chapter retraces that journey — from tally marks to zero, from debts to fractions, to the unbroken real number line.

Where numbers came from: counting, zero, and debt

Every family of number was invented to solve a problem the previous family could not. Natural numbers counted cattle. Zero named "nothing". Integers named debt. Each new set nests inside the next.

A history in bone. The Lebombo bone (~35,000 years old) has 29 notches; the Ishango bone (~20,000 BCE) groups tallies as 11, 13, 17, 19 — the prime numbers between 10 and 20. Natural numbers are at least that old.

Brahmagupta (628 CE) was the first to treat zero as a true number, defining it as $a-a=0$, and to give explicit rules for it: $a+0=a$, $a-0=a$, $a\times 0=0$.

He then extended the line leftward: **fortunes (dhana)** are positive, **debts (ṛiṇa)** are negative. Together with zero these form the **integers Z** (from German *Zahlen*, numbers).

Family	Symbol	What it contains	What it added
Natural numbers	N	1, 2, 3, 4, …	counting (one-to-one correspondence)
Whole numbers	W	0, 1, 2, 3, …	zero — Brahmagupta's Śhūnya
Integers	Z	…, −2, −1, 0, 1, 2, …	debt (ṛiṇa) alongside fortune (dhana)
Rational numbers	Q	p/q where p, q are integers and q ≠ 0	parts of a whole — fractions, positive and negative
Irrational numbers	—	cannot be written as p/q — e.g. √2, √3, π	lengths like the diagonal of a unit square
Real numbers	R	every rational and every irrational together	a number line with absolutely no gaps

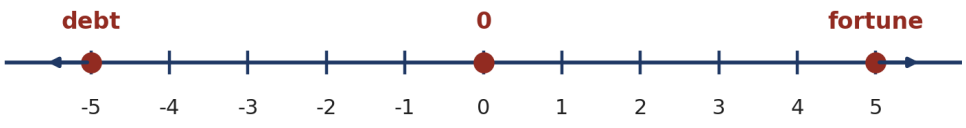

Fig. 3.1 — fortunes and debts on the integer number line

Brahmagupta's rules for integers

- A fortune plus a fortune is a fortune: $5+4=9$.
- A debt plus a debt is a debt: $(-5)+(-4)=-9$.
- The product of a debt and a fortune is a debt: $(-3)\times 4=-12$.
- The product of two debts is a fortune: $(-3)\times(-4)=12$ — removing 4 debts of ₹3 each leaves you ₹12 richer.

Rational numbers: filling the spaces between integers

A **rational number** is any number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ — the set is denoted $mathbb Q$.

Every integer is rational ($5 = \frac{5}{1}$), so N ⊂ W ⊂ Z ⊂ Q.

A rational number has **no unique representation**: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \dots$ are all equivalent. We usually quote the form in lowest terms, where p and q are co-prime.

Absolute value ¿ x ∨ ¿ is the distance of x from 0. For two rationals a and b, the distance between them on the line is ¿ $a - b$ ∨ ¿.

Density. Between any two rationals there is always another — take their average, $\frac{a+b}{2}$. Repeat forever and you get infinitely many rationals between any two, however close.

Arithmetic of rationals (Brahmagupta's laws)

- Addition/subtraction (same denominator): $\frac{a}{b} \pm \frac{c}{b} = \frac{a \pm c}{b}$
- Multiplication: $\frac{a}{b} \times \frac{c}{d} = \frac{a\,c}{b\,d}$
- Division: $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ (provided $c \neq 0$)
- Q is closed under $+, -, \times$, and under $\div$ as long as you never divide by zero.

Locating 3/4 on the number line

Represent $\frac{3}{4}$ on the number line.

Divide the unit interval between 0 and 1 into 4 equal parts (the denominator tells you how many parts).

Count 3 parts to the right of 0 (the numerator tells you how many steps).

Answer: the point $\frac{3}{4}$ lies three-quarters of the way from 0 to 1.

Irrational numbers: the diagonal that has no fraction

Baudhāyana (~800 BCE) met lengths that defied fractions while designing fire altars. A square of side 1 has diagonal d with $d^2 = 1^2 + 1^2 = 2$, so $d = \sqrt{2}$ — and $\sqrt{2}$ turns out **not** to be a ratio of integers.

Hippasus's proof (c. 400 BCE), by contradiction, shows $\sqrt{2}$ is irrational. Assume $\sqrt{2} = \frac{p}{q}$ in lowest terms. Squaring: $2q^2 = p^2$, so p^2 is even, so p is even — write $p = 2k$. Then $2q^2 = 4k^2$, so $q^2 = 2k^2$, so q is also even. But then p and q share the factor 2, contradicting "lowest terms". So $\sqrt{2}$ cannot be a fraction.

π is irrational too (proved by Lambert, 1761). Āryabhaṭa's $\frac{3927}{1250} = 3.1416$ is an excellent *approximation*, never an exact value — writing "$\pi = \frac{22}{7}$" is the most common error in this chapter.

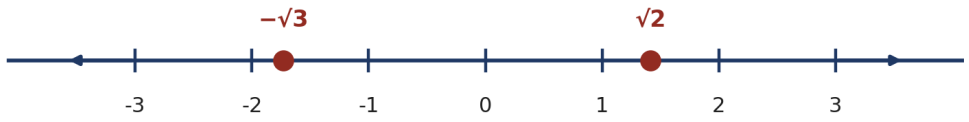

Fig. 3.2 — √2 and −√3 located on the number line

✏ Constructing an irrational length

- Mark O and A with $OA = 1$ unit; draw a perpendicular at A.
- Mark B on that perpendicular with $AB = 1$; then $OB = \sqrt{2}$ (Baudhāyana–Pythagoras).
- With centre O and radius OB, draw an arc to meet the number line at P: that point represents $\sqrt{2}$.

Decimal expansions: the test that never fails

You can tell a rational from an irrational just by looking at its decimal.

Predicting the type without dividing. Write $\frac{p}{q}$ in lowest terms. The decimal **terminates** exactly when the prime factors of q are only 2s, only 5s, or both — because then you can scale q up to a power of 10.

Why repeating decimals repeat: dividing by 7 can only ever leave remainders 1–6. Once a remainder recurs, the whole division loops.

Cyclic numbers. $\frac{1}{7} = 0.\overline{142857}$. Multiplying 142857 by 1–6 just cycles the same six digits: $142857 \times 3 = 428571$.

0.999… = 1. Let $x = 0.\overline{9}$. Then $10x = 9.\overline{9}$, so $10x - x = 9$, giving $9x = 9$ and $x = 1$. Not a trick — two names, one point.

Decimal behaves like…	Example	The number is
Terminating	0.375, 0.15	**Rational**
Non-terminating but recurring	0.6666… , 0.142857142857…	**Rational**
Non-terminating and non-recurring	1.41421356…, 3.14159265…	**Irrational**

Converting a general repeating decimal to p/q

Convert $2.3\overline{57}$ into the form $\frac{p}{q}$.

Let $x = 2.35757\ldots$ "35" is non-repeating (2 digits), "57" repeats (2 digits).

Multiply by 10^2 to clear the non-repeating part: $100x = 235.757\ldots$

Multiply again by 10^2 to shift one full repeating block: $10000x = 23575.757\ldots$

Subtract: $10000x - 100x = 23575.75\ldots - 235.75\ldots$, so $9900x = 23340$.

Answer: $x = \frac{23340}{9900} = \frac{1167}{495}$.

Surds and the laws of exponents

Real numbers written with root signs are called **surds**. For positive real a and b:

$$\sqrt{a}\sqrt{b}=\sqrt{ab}$$

$$\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$$

$$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$$

Rationalising the denominator removes the surd from the bottom of a fraction by multiplying top and bottom by the *conjugate* — the same expression with the middle sign flipped.

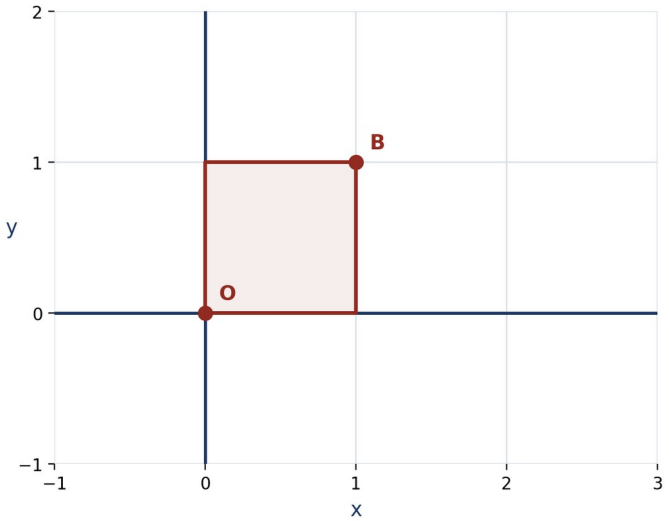

Fig. 3.3 — the unit square whose diagonal is √2

Laws of exponents for real numbers

- $a^m \cdot a^n = a^{m+n}$
- ¿
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $a^{1/n} = \sqrt[n]{a}$, and $a^0 = 1$ for any $a \neq 0$

Rationalising step by step

Rationalise $\dfrac{1}{\sqrt{5}-2}$.

The conjugate of $\sqrt{5}-2$ is $\sqrt{5}+2$.

$$\frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{\sqrt{5}+2}{¿¿}$$

¿ $\frac{\sqrt{5}+2}{5-4}=\sqrt{5}+2$

Check: the denominator is now a plain integer 1.

Part 2 · Worked Examples

Cover the right-hand column with a sheet of paper. Try the problem yourself first. Then uncover it one line at a time and check where your reasoning parted from the model.

Example 1 — Insert three rational numbers between $\frac{1}{2}$ and $\frac{3}{4}$

Step 1. Give both fractions a common denominator with enough room. Use 16: $\frac{1}{2}=\frac{8}{16}, \frac{3}{4}=\frac{12}{16}$.

Step 2. Read off numbers strictly between 8/16 and 12/16.

$\frac{9}{16}, \frac{10}{16}, \frac{11}{16}$

Step 3. Simplify: $\frac{10}{16}=\frac{5}{8}$.

Answer: $\frac{9}{16}, \frac{5}{8}, \frac{11}{16}$

> **Why this works**
>
> Any denominator bigger than the gap works. Creating space first, then reading off, is the whole method.

Example 2 — Prove that $\sqrt{3}$ is irrational

Step 1. Assume, for contradiction, $\sqrt{3}=\frac{p}{q}$ in lowest terms (p, q co-prime integers, $q \neq 0$).

Step 2. Square both sides: $3=\frac{p^2}{q^2}$, so $p^2=3q^2$.

Step 3. p^2 is a multiple of 3, so p must be a multiple of 3. Write $p=3k$.

Step 4. Substitute: $9k^2=3q^2$, so $q^2=3k^2$ — so q is also a multiple of 3.

Step 5. Both p and q share the factor 3, contradicting "lowest terms".

Answer: the assumption is false, so $\sqrt{3}$ is irrational.

> **Why this works**
>
> Exactly Hippasus's method for $\sqrt{2}$, with "even" replaced by "multiple of 3". The same idea proves $\sqrt{n}$ irrational for any non-perfect-square n.

Example 3 — Convert $0.1\overline{6}$ to the form $\frac{p}{q}$

Step 1. Let $x = 0.1666\ldots$ One digit ("1") is non-repeating, one digit ("6") repeats.

Step 2. Multiply by 10 to clear the non-repeating digit: $10x = 1.666\ldots$

Step 3. Multiply by 10 again to shift one repeating block: $100x = 16.666\ldots$

Step 4. Subtract: $100x - 10x = 16.666\ldots - 1.666\ldots$, so $90x = 15$.

Answer: $x = \frac{15}{90} = \frac{1}{6}$.

> **Why this works**
>
> The two-stage multiply (clear the lead-in, then shift one full cycle) is exactly the general method the chapter's summary table describes.

Part 3 · Guided Practice

The method is laid out for you. Fill in the blanks. If you can do these without looking back at Part 2, you are ready for Part 4.

1. Insert two rational numbers between $\frac{1}{4}$ and $\frac{1}{2}$.

Rewrite with denominator 8: $\frac{1}{4}=\frac{¿}{8}$ and $\frac{1}{2}=\frac{¿}{8}$

Only one gap, so scale up to 16: $\frac{¿}{16}$ and $\frac{¿}{16}$

Two numbers strictly between them: _____ and _____ .

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2. Show that $0.\overline{4}$ is rational by writing it as $\frac{p}{q}$.

Let $x=0.444\ldots$ One digit recurs, so multiply by _____ .

$10x=¿$ _____

Subtract: $¿_{x}=4$, so $x=¿$ _____ .

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3. Use Brahmagupta's rules to evaluate $(-8)\times(-7)$.

A debt times a debt is a _____ .

So the sign of the answer is _____ .

Magnitude: $8\times7=¿$ _____ , so the final answer is _____ .

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4. Rationalise the denominator of $\frac{2}{\sqrt{5}}$.

Multiply top and bottom by _____ .

$$\frac{2}{\sqrt{5}}\times\frac{¿}{¿=\frac{¿}{5}¿}$$

Final answer: _____

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5. Determine, without dividing, whether $\frac{7}{80}$ terminates.

Factorise the denominator: $80=2^{¿\times5^{¿¿}}$.

The prime factors are only _____ and _____ , so the decimal _____ (terminates / recurs).

Practice Set A · Fluency

Straight application of the method. Aim to get every one right, then aim to get them right quickly.

#	Question	Your working and answer
1	Is $\frac{-8}{1}$ a rational number, an integer, or both? Justify.	
2	Write three rational numbers between 2 and 3.	
3	State whether $0.\overline{8}$ is rational or irrational, and why.	
4	Simplify $\sqrt{72}$ into the form $a\sqrt{b}$ with b smallest possible.	
5	Convert $0.\overline{7}$ to the form $\frac{p}{q}$.	
6	Evaluate using Brahmagupta's rules: $(-15)\times 4$.	
7	Rationalise $\frac{1}{\sqrt{6}}$.	
8	Classify $\sqrt{49}$ as rational or irrational.	
9	Simplify $3^2 \cdot 3^5$ as a single power of 3.	
10	Without dividing, say whether $\frac{11}{40}$ terminates or recurs.	

Practice Set B · Reasoning

Two or more steps, or a step you have to choose for yourself. Show every line — marks live in the working.

#	Question	Your working and answer
1	Express $2.4\overline{5}$ in the form $\frac{p}{q}$.	
2	Rationalise the denominator of $\frac{1}{\sqrt{8}-\sqrt{6}}$.	
3	A trader takes a debt of ₹650, then earns a fortune of ₹900, then loses ₹300. Write this as an integer equation and find his final standing.	
4	Find three rational numbers strictly between $\frac{-1}{2}$ and $\frac{-1}{4}$.	
5	Simplify $(4+\sqrt{7})(4-\sqrt{7})$ and state whether the result is rational.	
6	Find the reciprocal of $\frac{2}{13}$ using long division and identify whether the block is cyclic (compare with $\frac{1}{7}$).	
7	If $\sqrt{2}=1.414$, find $\frac{3}{\sqrt{2}}$ correct to three decimal places.	
8	A tailor has $15\frac{3}{4}$ metres of silk; each kurta needs $2\frac{1}{4}$ metres. How many whole kurtas can he make?	

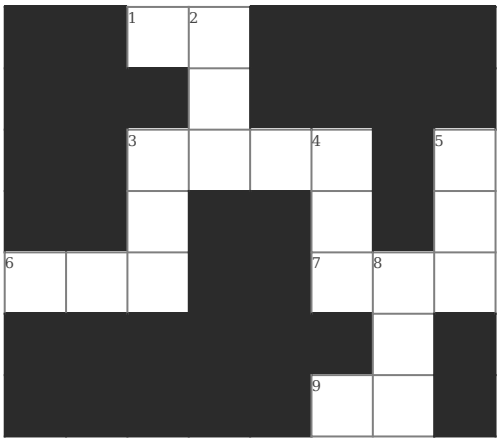
Practice Set C · Challenge

Harder, and deliberately so. If one defeats you, write down exactly where you got stuck. That sentence is worth more than a copied answer.

#	Question	Your working and answer
1	Prove that $\sqrt{5}$ is irrational, adapting Hippasus's method.	
2	Show that $0.\overline{9}=1$ using algebra, and explain in one line why this is not a trick.	
3	If $x=2+\sqrt{3}$, find the value of $x+\frac{1}{x}$.	
4	Is the sum of two irrational numbers always irrational? Give a proof or a counter-example.	
5	A rational number in lowest terms has denominator $2^3 \times 5^2$. How many decimal places will its expansion have? Explain using the chapter's "power of 10" argument.	

Part 7 · The Crossnumber

Every answer is a number. One digit per square. Because answers cross, a wrong calculation simply will not fit — this puzzle marks itself.



ACROSS

1. 3^4 *(2 digits)*

3. 2^{10} *(4 digits)*

6. 3^6 *(3 digits)*

7. 5^3 *(3 digits)*

9. The square of the number of years the Ishango bone predates 1900, in thousands (approx.) *(2 digits)*

DOWN

2. A power of 10 that clears two repeating digits *(3 digits)*

3. 13^2 — a Pythagorean-triple square *(3 digits)*

4. 21^2 *(3 digits)*

5. 5^4 *(3 digits)*

8. 2^8 *(3 digits)*

Bonus questions that did not fit the grid — work them out anyway: 7^3; The square of 12

Part 8 · Number Search & Magic Square

Work out each answer, then find it hidden in the digit grid — across, down, diagonally or backwards.

6	1	8	7	6	9	2	6	8	5	0	6
5	9	0	5	3	0	0	7	5	9	1	8
0	1	6	3	1	0	8	3	3	8	5	0
4	4	6	1	4	1	1	2	5	3	9	8
8	4	2	8	2	4	4	5	5	2	5	1
9	9	5	4	5	0	0	5	6	5	2	0
7	4	2	0	1	4	7	0	4	3	4	4
1	7	0	9	9	8	2	6	4	7	9	6
7	2	3	4	2	1	3	0	1	8	4	5
2	5	0	2	6	1	3	4	9	4	6	7
9	1	8	1	8	3	4	3	6	8	4	9
2	4	0	7	9	6	3	7	1	3	8	6

☐ 1024	☐ 169	☐ 441	☐ 125	☐ 256
☐ 343	☐ 729	☐ 625	☐ 144	☐ 100
☐ 81	☐ 36			

Magic Square — every row, column and diagonal adds to the same total.

8		
3	**5**	
4		**2**

Magic constant: 15. Fill the empty cells.

Part 9 · Spot the Mistake

Each solution below reaches a wrong answer, and exactly one step is to blame. Find the first wrong step and rewrite it correctly. Marking other people's work is the fastest way to stop making their mistakes.

🔍 Spot the Mistake

Converting $0.\overline{45}$ to a fraction

Step 1. Let $x = 0.454545\ldots$

Step 2. Two digits recur, so multiply by 10: $10x = 4.5454\ldots$

Step 3. Subtract: $9x = 4.0909\ldots$, so $x = 0.4545\ldots$

_____ The first wrong step is Step _____ . What should it say?

_____

🔍 Spot the Mistake

Rationalising $\dfrac{1}{\sqrt{7}-2}$

Step 1. Multiply top and bottom by the conjugate $\sqrt{7}+2$.

Step 2. Denominator becomes ¿.

Step 3. So the answer is $\dfrac{\sqrt{7}-2}{3}$.

_____ The first wrong step is Step _____ . What should it say?

_____

🔍 Spot the Mistake

Applying Brahmagupta's rule for two debts

Step 1. $(-6)\times(-9)$ involves two debts.

Step 2. A debt times a debt is a debt, so the answer is negative.

Step 3. $6\times 9 = 54$, so the final answer is -54.

_____ The first wrong step is Step _____ . What should it say?

_____

Part 10 · Quick-Fire Round

Match each expression to its simplified/decimal form

Draw a line from each item on the left to its match on the right.

#	Left	Right (shuffled)
1	$\sqrt{72}$	4
2	$\sqrt{45}$	$6\sqrt{2}$
3	$0.\overline{6}$	$0.\overline{142857}$
4	$0.\overline{45}$	$\frac{5}{11}$
5	$\frac{1}{7}$	a terminating decimal
6	$(-4) \times (-9)$	$3\sqrt{5}$
7	$(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})$	36 (a fortune)
8	$\frac{7}{80}$	$\frac{2}{3}$

True or False Sprint

Everyone stands. Teacher reads a statement. True → stay standing. False → sit down.

#	Statement	Your call
1	Every integer is a rational number.	☐ True ☐ False
2	Every rational number is an integer.	☐ True ☐ False
3	Zero was treated as a number and given arithmetic rules by Brahmagupta.	☐ True ☐ False
4	π is exactly equal to $\frac{22}{7}$.	☐ True ☐ False
5	The product of two debts, in Brahmagupta's language, is a fortune.	☐ True ☐ False
6	A terminating decimal is always rational.	☐ True ☐ False
7	A non-terminating recurring decimal is irrational.	☐ True ☐ False
8	Between any two rational numbers there are infinitely many rational numbers.	☐ True ☐ False

#	Statement	Your call
9	$\sqrt{16}$ is an irrational number.	☐ True ☐ False
10	The decimal expansion of $\frac{1}{7}$ shows cyclic digit patterns.	☐ True ☐ False
11	$0.\overline{9}$ is strictly less than 1.	☐ True ☐ False
12	A rational number whose denominator (in lowest terms) has only 2 and 5 as prime factors will have a terminating decimal.	☐ True ☐ False

Two Truths and a Lie

Two statements are true, one is false. Circle the lie — then correct it.

Set 1

(a) Every whole number is an integer.
(b) Every integer is a whole number.
(c) Every natural number is a whole number.

The lie is _____ . Corrected: _____

Set 2

(a) Brahmagupta defined zero as $a-a=0$.
(b) Brahmagupta called negative numbers "fortunes" (dhana).
(c) Brahmagupta called positive numbers "fortunes" (dhana).

The lie is _____ . Corrected: _____

Set 3

(a) $\sqrt{2}$ is irrational.
(b) All non-terminating decimals are irrational.
(c) $0.\overline{3}$ is rational.

The lie is _____ . Corrected: _____

Part 11 · The Escape Challenge

Five gates, each a calculation. Each gives you one digit. Collect all five to unlock the exit code.

🔒 GATE 1 — The Fortune Gate

Evaluate $(-5) \times (-6)$ using Brahmagupta's rule for two debts.

Digit: ☐

🔒 GATE 2 — The Recurring Gate

Write $0.\overline{4}$ as $\frac{p}{q}$ in lowest terms. Take the value of p.

Digit: ☐

🔒 GATE 3 — The Root Gate

Evaluate $\sqrt{81}$.

Digit: ☐

🔒 GATE 4 — The Surd Gate

Evaluate $(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})$.

Digit: ☐

🔒 GATE 5 — The Family Gate

How many number families are listed in the Learn First table, from natural numbers to real numbers?

Digit: ☐

EXIT CODE: ☐ ☐ ☐ ☐ ☐

Formula Sheet

Cut this page out and keep it. Everything in this chapter, on one page.

Rational number

$$\frac{p}{q}, p, q \in Z, q \neq 0$$

Zero (Brahmagupta)

$$a+0=a, a-0=a, a \times 0=0$$

Two debts make a fortune

$$(-a) \times (-b)=a\,b$$

Average lies between

$$a<\frac{a+b}{2}<b (a<b)$$

Product of surds

$$\sqrt{a}\sqrt{b}=\sqrt{a\,b}$$

Difference of squares

$$(\sqrt{a}+\sqrt{b})(\sqrt{a}-\sqrt{b})=a-b$$

Laws of exponents

$$a^m \cdot a^n = a^{m+n}, ¿$$

Fractional index

$$a^{1/n}=\sqrt[n]{a}$$

Full Solutions

Not just answers — working. Compare your method with this one; if they differ but both reach the right answer, make sure you understand why yours also works.

Part 3 · Guided Practice

1. 5/16 and 6/16 = 3/8 (any two of 5/16, 6/16, 7/16) · **2.** 10; 10x = 4.444...; 9x = 4, x = 4/9 · **3.** fortune; positive; 56; +56 · **4.** √5; 2√5/5 · **5.** 2^4 x 5^1; 2 and 5; terminates

Practice Set A

1. Both — every integer, including -8, is also rational since $-8=\frac{-8}{1}$.

2. Any three of $\frac{9}{4}, \frac{5}{2}, \frac{11}{4}$.

3. Rational — a non-terminating recurring decimal always equals a fraction.

4. $\sqrt{72}=\sqrt{36\times 2}=6\sqrt{2}$.

5. $x=0.777\ldots;\ 10x=7.777\ldots;\ 9x=7;\ x=\frac{7}{9}$.

6. A debt times a fortune is a debt: $(-15)\times 4=-60$.

7. $\frac{1}{\sqrt{6}}\times\frac{\sqrt{6}}{\sqrt{6}}=\frac{\sqrt{6}}{6}$.

8. Rational — 49 is a perfect square, so $\sqrt{49}=7$.

9. $3^2\cdot 3^5=3^7=2187$.

10. $40=2^3\times 5$ — only 2s and 5s, so it **terminates**.

Practice Set B

1. $x=2.4555\ldots;\ 10x=24.555\ldots;\ 100x=245.555\ldots;\ 100x-10x=221;\ x=\frac{221}{90}$.

2. Multiply by $\frac{\sqrt{8}+\sqrt{6}}{\sqrt{8}+\sqrt{6}}$: denominator ¿$8-6=2$, giving $\frac{\sqrt{8}+\sqrt{6}}{2}=\frac{2\sqrt{2}+\sqrt{6}}{2}$.

3. $-650+900-300=-50$; he ends with a net debt of ₹50.

4. Any three of $\frac{-7}{16}, -\frac{3}{8}, -\frac{5}{16}$ (between $-1/2=-8/16$ and $-1/4=-4/16$).

5. $16-7=9$ — rational.

6. $\frac{2}{13}=0.\overline{153846}$ — a 6-digit repeating block; it is cyclic in the same way as $\frac{1}{7}$ (its multiples cyclically permute the digits).

7. $\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}=\frac{3\times 1.414}{2}=2.121$.

8. $15\frac{3}{4} = \frac{63}{4}$ metres; $2\frac{1}{4} = \frac{9}{4}$ metres per kurta; $\frac{63}{4} \div \frac{9}{4} = 7$ kurtas exactly.

Practice Set C

1. Assume $\sqrt{5} = \frac{p}{q}$ in lowest terms. Then $p^2 = 5q^2$, so p is a multiple of 5; write $p = 5k$, giving $25k^2 = 5q^2$, so $q^2 = 5k^2$ and q is also a multiple of 5 — contradicting "lowest terms". Hence $\sqrt{5}$ is irrational.

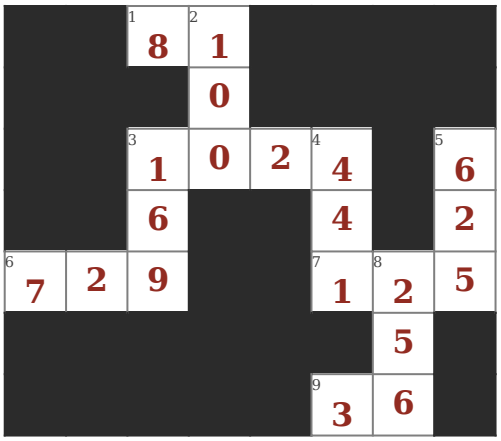
2. Let $x = 0.999\ldots$; $10x = 9.999\ldots$; $9x = 9$; $x = 1$. Not a trick — $0.\overline{9}$ and 1 are two decimal names for the same point on the line.

3. $\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = 2-\sqrt{3}$ since $4-3=1$. So $x + \frac{1}{x} = (2+\sqrt{3}) + (2-\sqrt{3}) = 4$.

4. No in general — but it can be. Counter-example where it fails: $\sqrt{2} + (-\sqrt{2}) = 0$, which is rational, even though both terms are irrational.

5. $50 = 2 \times 5^2$ — only 2s and 5s — so it terminates, and to reach a power of 10 we need 10^3 (since the highest power present is 5^2 needing a matching 2^2, and the 2^1 present needs boosting to 2^3): the expansion has **3** decimal places.

Part 7 · Crossnumber — Solved Grid



Across: 1 = 81 · 3 = 1024 · 6 = 729 · 7 = 125 · 9 = 36
Down: 2 = 100 · 3 = 169 · 4 = 441 · 5 = 625 · 8 = 256

Part 8 · Number Search — Solved Grid

6	1	8	7	**6**	9	2	6	8	5	0	6
5	9	0	5	**3**	**0**	0	7	5	9	**1**	**8**
0	**1**	6	3	1	**0**	8	3	3	8	5	0
4	**4**	**6**	1	4	**1**	**1**	**2**	**5**	3	9	8

8	**4**	**2**	8	2	4	4	5	5	2	5	1
9	9	**5**	4	5	0	0	5	**6**	**5**	**2**	0
7	**4**	**2**	**0**	**1**	4	7	0	**4**	3	4	4
1	7	0	9	9	8	2	6	**4**	7	9	6
7	2	3	4	2	1	3	0	**1**	8	4	5
2	5	0	2	6	1	**3**	4	**9**	4	6	7
9	1	8	1	8	3	**4**	3	**6**	8	4	9
2	4	0	7	9	6	**3**	7	**1**	3	8	6

Part 8 · Magic Square — Solved (constant 15)

8	**1**	**6**
3	**5**	**7**
4	**9**	**2**

Part 9 · Spot the Mistake

Problem	First wrong step	It should say
Converting $0.\overline{45}$ to a fraction	Step 2	Two recurring digits need a multiplier of **100**, not 10: $100x = 45.4545\ldots$, giving $99x = 45$ and $x = \frac{45}{99} = \frac{5}{11}$.
Rationalising $\frac{1}{\sqrt{7}-2}$	Step 2	The **numerator** must also carry the conjugate: the answer is $\frac{\sqrt{7}+2}{3}$, not $\frac{\sqrt{7}-2}{3}$.
Applying Brahmagupta's rule for two debts	Step 1	A debt times a debt is a **fortune** (positive): $(-6)\times(-9) = +54$, not -54.

Part 10 · Matching

#	Left	Correct match
1	$\sqrt{72}$	$6\sqrt{2}$

#	Left	Correct match
2	$\sqrt{45}$	$3\sqrt{5}$
3	$0.\overline{6}$	$\frac{2}{3}$
4	$0.\overline{45}$	$\frac{5}{11}$
5	$\frac{1}{7}$	$0.\overline{142857}$
6	$(-4) \times (-9)$	36 (a fortune)
7	$(\sqrt{6}+\sqrt{2})(\sqrt{6}-\sqrt{2})$	4
8	$\frac{7}{80}$	a terminating decimal

Part 10 · True or False

1-T · 2-F · 3-T · 4-F · 5-T · 6-T · 7-F · 8-T · 9-F · 10-T · 11-F · 12-T

Part 10 · Two Truths and a Lie

Set	The lie	Correction
1	b	Negative integers such as -3 are not whole numbers.
2	b	Dhana (fortune) meant positive numbers; ṛiṇa (debt) meant negative numbers — statement (b) has them swapped.
3	b	Non-terminating *recurring* decimals are rational — only non-recurring ones are irrational.

Part 11 · The Escape Challenge

EXIT CODE: 30 · 4 · 9 · 4 · 6

(Answer: 30) (Answer: 4) (Answer: 9) (Answer: 4) (Answer: 6)

From tally marks on bone to Brahmagupta's zero, from dense fractions to the unbroken real line — you now command every number this chapter uncovered. The universe really is written in their language.